

## UNIT I - BASIC CONCEPTS OF MODELING

### **3 phase synchronous machine with and without damper bars and 3 - phase induction machine**

Introduction:

It was shown in an earlier chapter that an alternator driven at a constant speed produces an alternating voltage at a fixed frequency dependent on the number of poles in the machine. A machine designed to be connected to the supply and run at synchronous speed is referred to as a synchronous machine. The description applies to both motors and generators. A synchronous condenser is a special application of a synchronous motor. While the synchronous motor has only one generally used name, the synchronous generator is on occasion referred to as an alternator or as an a.c. generator. The term alternator has been used in previous chapters and will be used in this chapter but it should be remembered that other terms are in use. In general, the principles of construction and operation are similar for both alternators and generators, just as there were basic similarities between d.c. motors and generators. While alternators were once seldom seen outside power houses and whole communities were supplied from a central source, there is now an expanding market for smaller sized alternators suitable for the provision of power for portable tools. Today, with the growth in computer control, there is a further need for standby generating plant to ensure a continuity of supply to prevent a loss of data from computer memories. So much information is being stored in computers today that even brief interruptions to the power supplies can have serious consequences on the accuracy and extent of information stored.

#### **Alternators**

The three-phase synchronous machine has two main windings:

1. a three-phase a.c. winding;
2. another winding carrying d.c.

In the majority of cases, the rotor has the d.c. winding and the stator the a.c. winding. An alternator with a rotating a.c. winding and a stationary d.c. winding, while suitable for smaller outputs, is not satisfactory for the larger outputs required at power stations. With these machines the output can be in megawatts; a value too large to be handled with brushes and slip rings. Because the terminal voltages range up to 33 kV, the only satisfactory construction is to have the a.c. windings stationary and to supply the rotor with d.c. This arrangement has the following advantages: 1. extra winding space for the a.c. windings; 2. easier to insulate for higher voltages; 3. simple, strong rotor construction; 4. lower voltages and currents in the rotating windings; 5. the high current windings have solid connections to the "outside" circuit; 6. better suited to the higher speeds (and smaller number of poles) of turbine drives.

#### **Stator**

The stator of the three-phase synchronous machine consists of a slotted laminated core into which the stator winding is fitted. The stator winding consists of three separate windings physically displaced from each other by  $120^\circ$ . Each phase winding has a number of coils connected in series to form a definite number of magnetic poles. A four-pole machine, for example, has four groups of coils per phase or four "pole-phase groups". The ends of the three phase windings are connected in either star or delta to the external circuit. Details of phase windings for a three-phase machine were shown in Chapter 10 to consist of three identical windings symmetrically distributed around the stator.

#### **Prime movers**

##### **Low speed**

Most diesel engines used as prime movers for alternators operate within the speed range of 50 r/min and this necessitates the use of rotors with pairs of poles.

Hydroelectric turbines have water-driven in which operate at low speeds, consequently they all rotors with many poles. While the diesel-driven alt usually has its shaft in the horizontal plane hydroelectric unit has its shaft in the vertical plane. The method of construction means that special thrust bearings have to be fitted to take the end thrust of the 1 component.

## High speed

Turbine prime movers, whether steam or gas, efficiently at speeds in the vicinity of 3000 r / n alternator driven by a turbine and producing a fn of 50 Hz at 3000 r /min must consist of only two In Chapter 6 the relationship between frequency and the number of poles was shown to

$$f = \frac{np}{120}$$

By transposition

$$n = \frac{120f}{p}$$

where  $n = r/min$

$f =$  frequency in hertz

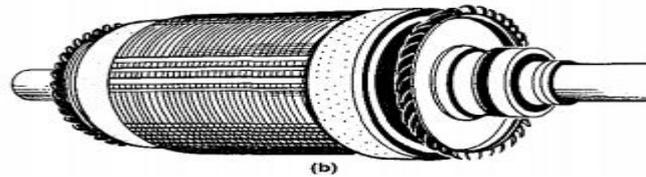
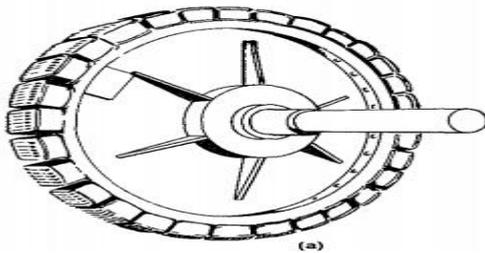
$p =$  number of poles

For a large-diameter rotor of twenty-four poles at 50 Hz

$$n = \frac{120 \times 50}{24} = 250 \text{ r/min}$$

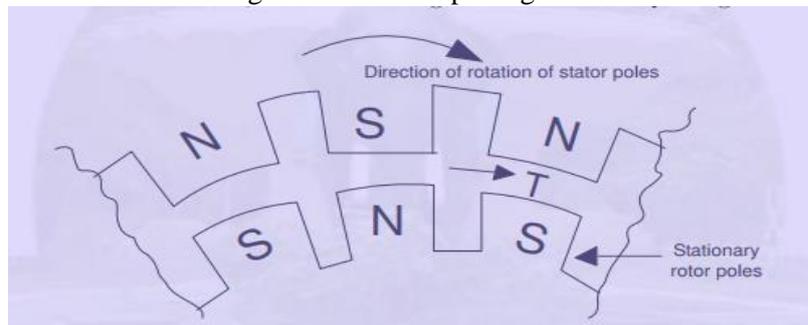
For a turbine-type rotor of two poles at 50 Hz

$$n = \frac{120 \times 50}{2} = 3000 \text{ r/min}$$



## Synchronous motors

Principle of operation In order to understand the principle of operation of a synchronous motor, let us examine what happens if we connect the armature winding (laid out in the stator) of a 3-phase synchronous machine to a suitable balanced 3-phase source and the field winding to a D.C source of appropriate voltage. The current flowing through the field coils will set up stationary magnetic poles of alternate North and South. ( for convenience let us assume a salient pole rotor, as shown in Fig. 50). On the other hand, the 3-phase currents flowing in the armature winding produce a rotating magnetic field rotating at synchronous speed. In other words there will be moving North and South poles established in the stator due to the 3-phase currents i.e at any location in the stator there will be a North pole at some instant of time and it will become a South pole after a time period corresponding to half a cycle. (after a time =  $1/2f$ , where  $f =$  frequency of the supply). Let us assume that the stationary South pole in the rotor is aligned with the North pole in the stator moving in clockwise direction at a particular instant of time, as shown in Fig. 50. These two poles get attracted and



try to maintain this alignment ( as per lenz's law) and hence the rotor pole tries to follow the stator pole as the conditions are suitable for the production of torque in the clockwise direction. However the rotor cannot move instantaneously due to its mechanical inertia, and so it needs sometime to move. In the mean time, the stator pole would quickly (a time duration corresponding to half a cycle) change its polarity and becomes a South pole. So the force of attraction will no longer be present and instead the like poles experience a force of repulsion as shown in Fig. 51. In other words, the conditions are now suitable for the

Force of repulsion between stator poles and rotor poles - resulting in production of torque in anticlockwise direction production of torque in the anticlockwise direction. Even this condition will not last longer as the stator pole would again change to North pole after a time of  $1/2f$ . Thus the rotor will experience an alternating force

which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to  $1/2f$  seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only. On the contrary if the rotor is brought to near synchronous speed by some external means say a small motor (known as pony motor-which could be a D.C or AC induction rotor) mounted on the same shaft as that of the rotor, the rotor poles get locked to the unlike poles in the stator and the rotor continues to run at the synchronous speed even if the supply to the pony motor is disconnected. Thus the synchronous rotor cannot start rotating on its own or usually we say that the synchronous rotor has no starting torque. So, some special provision has to be made either inside the machine or outside of the machine so that the rotor is brought to near about its synchronous speed. At that time, if the armature is supplied with electrical power, the rotor can pull into step and continue to operate at its synchronous speed. Some of the commonly used methods for starting synchronous rotor are described in the following section.

### Methods of starting synchronous motor

Basically there are three methods that are used to start a synchronous motor:

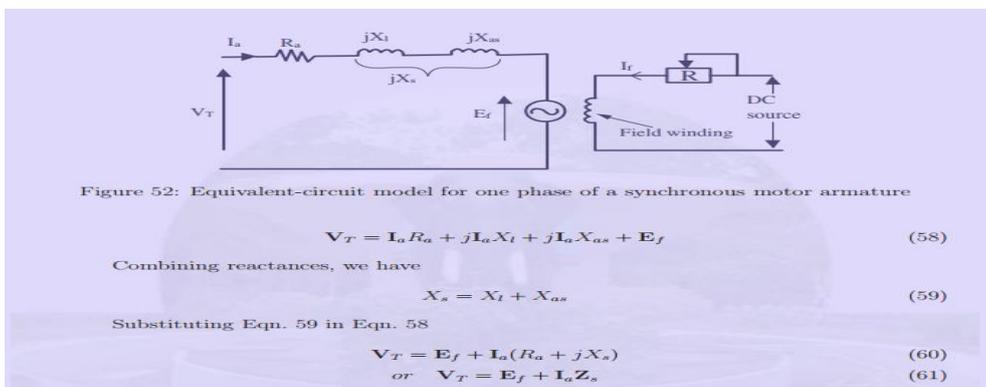
- To reduce the speed of the rotating magnetic field of the stator to a low enough value that the rotor can easily accelerate and lock in with it during one half-cycle of the rotating magnetic field's rotation. This is done by reducing the frequency of the applied electric power. This method is usually followed in the case of inverter-fed synchronous motor operating under variable speed drive applications.
- To use an external prime mover to accelerate the rotor of synchronous motor near to its synchronous speed and then supply the rotor as well as stator. Ofcourse care should be taken to ensure that the direction of rotation of the rotor as well as that of the rotating magnetic field of the stator are the same. This method is usually followed in the laboratory- the synchronous machine is started as a generator and is then connected to the supply mains by following the synchronization or paralleling procedure. Then the power supply to the prime mover is disconnected so that the synchronous machine will continue to operate as a motor.
- To use damper windings or amortisseur windings if these are provided in the machine. The damper windings or amortisseur windings are provided in most of the large synchronous motors in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load. Each of these methods of starting a synchronous motor are described below in detail.

### Behavior of a synchronous motor

The behavior of a synchronous motor can be predicted by considering its equivalent circuit on similar lines to that of a synchronous generator as described below.

Equivalent circuit model and phasor diagram of a synchronous motor

The equivalent-circuit model for one armature phase of a cylindrical rotor three phase synchronous motor is shown in Fig. 52 exactly similar to that of a synchronous generator except that the current flows in to the armature from the supply. All values are given per phase. Applying Kirchoff's voltage law to Fig. 52,



where:  $R_a$  = armature resistance ( $\Omega$ /phase)

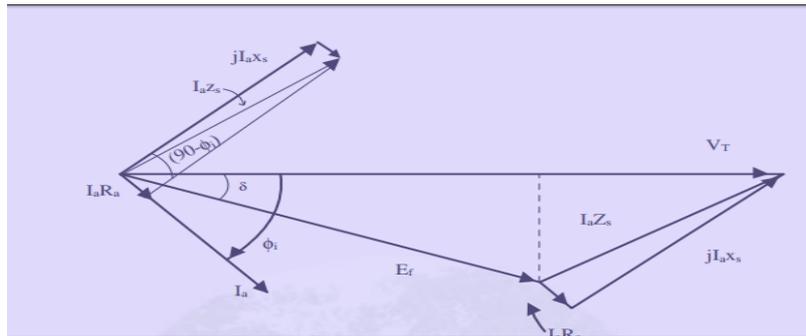
$X_l$  = armature leakage reactance ( $\Omega$ /phase)

$X_s$  = synchronous reactance ( $\Omega$ /phase)

$Z_s$  = synchronous impedance ( $\Omega$ /phase)

$V_T$  = applied voltage/phase (V)

$I_a$  = armature current/phase(A)



A phasor diagram shown in Fig. 53, illustrates the method of determining the counter EMF which is obtained from the phasor equation;

$$E_f = V_T - I_a Z_s$$

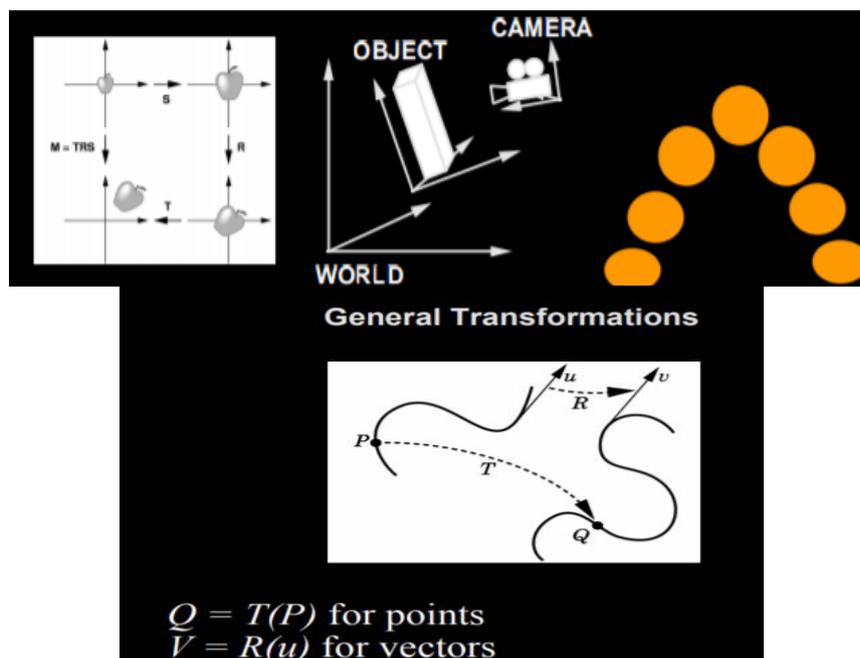
The phase angle  $\delta$  between the terminal voltage  $V_T$  and the excitation voltage  $E_f$  in Fig. 53 is usually termed the torque angle. The torque angle is also called the load angle or power angle.

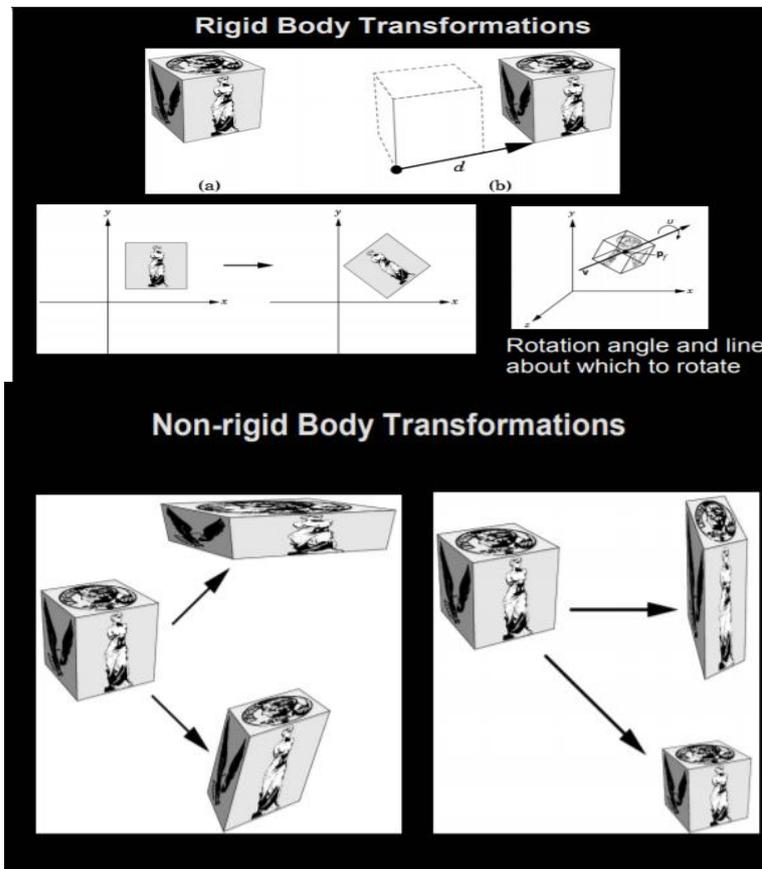
## UNIT II - REFERENCE FRAME THEORY

Transformation to obtain constant matrices:

Uses of Transformations

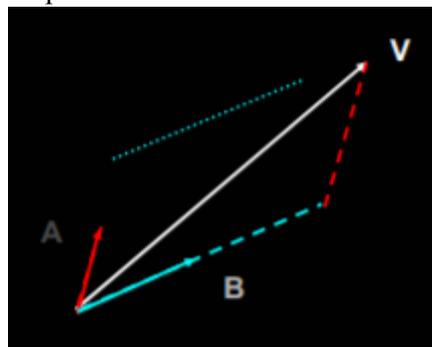
- Modeling transformations
  - build complex models by positioning simple components
  - transform from object coordinates to world coordinates
- Viewing transformations
  - placing the virtual camera in the world
  - i.e. specifying transformation from world coordinates to camera coordinates
- Animation
  - vary transformations over time to create motion





**Background Math: Linear Combinations of Vectors**

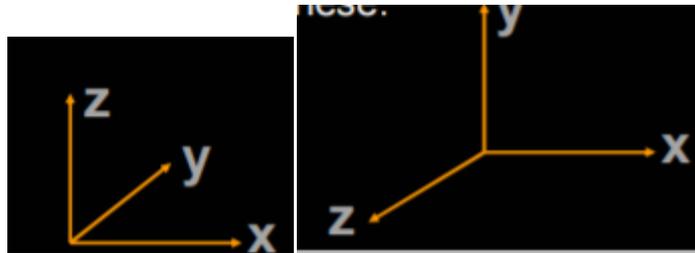
- Given two vectors, A and B, walk any distance you like in the A direction, then walk any distance you like in the B direction
- The set of all the places (vectors) you can get to this way is the set of linear combinations of A and B.
- A set of vectors is said to be linearly independent if none of them is a linear combination of the others.



$$V = v_1A + v_2B, (v_1, v_2) \in \mathbb{R}^2$$

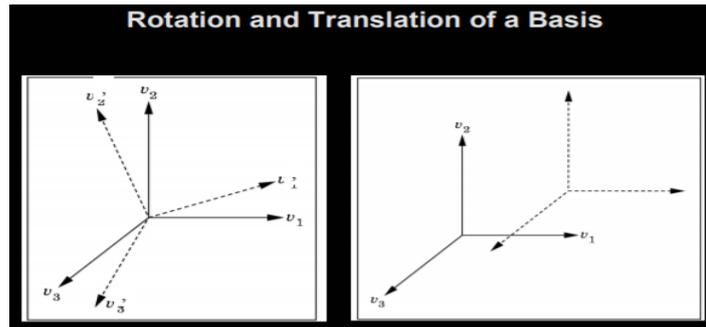
**Bases**

- A basis is a linearly independent set of vectors whose combinations will get you anywhere within a space, i.e. span the space
- $n$  vectors are required to span an  $n$ -dimensional space
- If the basis vectors are normalized and mutually orthogonal the basis is orthonormal
- There are lots of possible bases for a given vector space; there's nothing special about a particular basis—but our favorite is probably one of these



### Vectors Represented in a Basis

- Every vector has a unique representation in a given basis
- the multiples of the basis vectors are the vector's components or coordinates
- changing the basis changes the components, but not the vector
- $V = v_1E_1 + v_2E_2 + \dots + v_nE_n$  The vectors  $\{E_1, E_2, \dots, E_n\}$  are the basis. The scalars  $(v_1, v_2, \dots, v_n)$  are the components of  $V$  with respect to that basis.



### Linear and Affine Maps

- A function (or map, or transformation)  $F$  is linear if for all vectors  $A$  and  $B$ , and all scalars  $k$ .
- Any linear map is completely specified by its effect on a set of basis vectors:
- A function  $F$  is affine if it is linear plus a translation
- Thus the 1-D transformation  $y=mx+b$  is not linear, but affine
- Similarly for a translation and rotation of a coordinate system
- Affine transformations preserve lines

$$F(A+B) = F(A) + F(B) \quad F(kA) = k F(A)$$

$$V = v_1E_1 + v_2E_2 + v_3E_3$$

$$F(V) = F(v_1E_1 + v_2E_2 + v_3E_3) = F(v_1E_1) + F(v_2E_2) + F(v_3E_3) = v_1F(E_1) + v_2F(E_2) + v_3F(E_3)$$

### Transforming a Vector

• The coordinates of the transformed basis vector (in terms of the original basis vectors):

$$F(E_1) = f_{11}E_1 + f_{21}E_2 + f_{31}E_3$$

$$F(E_2) = f_{12}E_1 + f_{22}E_2 + f_{32}E_3$$

$$F(E_3) = f_{13}E_1 + f_{23}E_2 + f_{33}E_3$$

• The transformed general vector  $V$  becomes:

$$F(V) = v_1F(E_1) + v_2F(E_2) + v_3F(E_3)$$

$$= (f_{11}E_1 + f_{21}E_2 + f_{31}E_3)v_1 + (f_{12}E_1 + f_{22}E_2 + f_{32}E_3)v_2 + (f_{13}E_1 + f_{23}E_2 + f_{33}E_3)v_3$$

$$= (f_{11}v_1 + f_{12}v_2 + f_{13}v_3)E_1 + (f_{21}v_1 + f_{22}v_2 + f_{23}v_3)E_2 + (f_{31}v_1 + f_{32}v_2 + f_{33}v_3)E_3$$

and its coordinates (still w.r.t.  $E$ ) are

$$\tilde{v}_1 = (f_{11}v_1 + f_{12}v_2 + f_{13}v_3)$$

$$\tilde{v}_2 = (f_{21}v_1 + f_{22}v_2 + f_{23}v_3)$$

$$\tilde{v}_3 = (f_{31}v_1 + f_{32}v_2 + f_{33}v_3)$$

or just  $v_i = \sum_j f_{ij}v_j$  The matrix multiplication formula!

### Matrices to the Rescue

An  $n \times n$  matrix  $F$  represents a linear function in  $n$  dimensions

- $i$ -th column shows what the function does to the corresponding basis vector
- Transformation = linear combination of columns of  $F$  – first component of the input vector scales first column of the matrix – accumulate into output vector – repeat for each column and component
- Usually compute it a different way: – dot row  $i$  with input vector to get component  $i$  of output vector

$$\begin{Bmatrix} \tilde{v}_1 \\ \tilde{v}_2 \\ \tilde{v}_3 \end{Bmatrix} = \begin{Bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{Bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \quad v_i = \sum_j f_{ij}v_j$$

### Three Phase to Two Phase Transformation

This transformation could also be thought of as a transformation from three (abc) axes to three new (dq0) axes for uniqueness of the transformation from one set of axes to another set of axes, including unbalances in the abc variables requires three variables such as the dq0.[4] The reason for this is that it is easy to convert from three abc variables to dq variables if the abc variables have an inherent relationship among themselves, such as the equal phase displacement and magnitude. Therefore, in such a case there are only two independent variables in a,b,c: the third is a dependent variable obtained is unique under that circumstance This paper the variables have no such inherent relationship, then there are three distinct and independent variables: Hence the third variable cannot be recovered from the knowledge of the other two variables only[5]. It also means that they are not recoverable from two variables dq but require another variable only[5]. It also means that they are not recoverable from two variables dq but require another variable such as the zero sequence component, to recover the [7] abc variables from the variables.

### THREE PHASE(a,b,c) to TWO PHASE(d,q,0) TRANSFORMATION

In electrical engineering, direct-quadrature-zero (abc) transformation or zero-direct-quadrature (dq0) transformation is a mathematical transformation that rotates the reference frame of three-phase systems in an effort to simplify the analysis of three-phase circuits. In the case of balanced three-phase circuits, application of the dq0 transform reduces the three AC quantities to two DC quantities. Simplified calculations can then be carried out on these DC quantities before performing the inverse transform to recover the actual three-phase AC results. It is often used in order to simplify the analysis of three-phase synchronous machines or to simplify calculations for the control of three-phase inverters. The power-invariant, right-handed dq0 transform applied to any three-phase quantities (e.g. voltages,

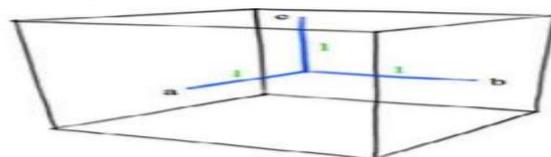
$$x_{dq0} = K x_{abc} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

The inverse transform is:

$$x_{abc} = K^{-1} x_{dq0} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \frac{\sqrt{2}}{2} \\ \cos(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \\ \cos(\theta + \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix}$$

### Geometric Interpretation

The dq0 transformation is two sets of axis rotations in sequence. We can begin with a 3D space where a, b, and c are orthogonal axes.



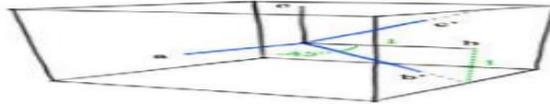
If we rotate about the axis by  $-45^\circ$ , we get the following rotation matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\pi/4) & \sin(-\pi/4) \\ 0 & -\sin(-\pi/4) & \cos(-\pi/4) \end{bmatrix}$$

which resolves to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

With this rotation, the axes look like



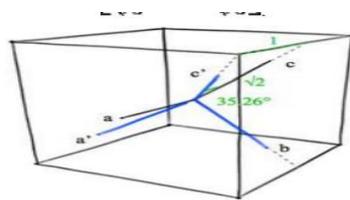
Then we can rotate about the new b axis by

$$\phi = \cos^{-1} \sqrt{\frac{2}{3}}$$

$$\begin{bmatrix} \cos(\phi) & 0 & -\sin(\phi) \\ 0 & 1 & 0 \\ \sin(\phi) & 0 & \cos(\phi) \end{bmatrix}$$

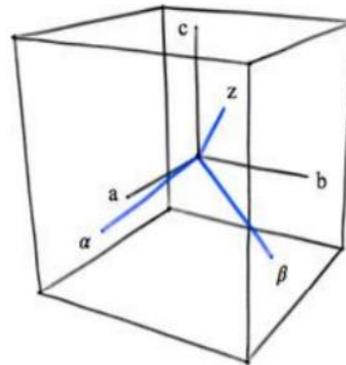
Which resolves to

$$\begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix}$$



When these two matrices are multiplied, we get the Clarke transformation matrix C:

$$\sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$



This is the first of the two sets of axis rotations. At this point, we can relabel the rotated a, b, and c axes as  $\alpha$ ,  $\beta$ , and z. This first set of rotations places the z axis an equal distance away from all three of the original a, b, and c axes. In a balanced system, the values on these three axes would always balance each other in such a way that the z axis value would be zero. This is one of the core values of the dqo transformation; it can reduce the number relevant variables in the system. The second set of axis rotations is very simple. In electric systems, very often the a, b, and c values are oscillating in such a way that the net vector is spinning. In a balanced system, the vector is spinning about the z axis. Very often, it is helpful to rotate the reference frame such that the majority of the changes in the abc values, due to this spinning, are canceled out and any finer variations in become more obvious. So, in addition to the Clarke transform, the following axis rotation is applied about the z axis:

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying this matrix by the Clarke matrix results in the dqo transform:

$$\sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

The dqo transformation can be thought of in geometric terms as the projection of the three separate sinusoidal phase quantities onto two axes rotating with the same angular velocity as the sinusoidal phase quantities. The two axes are called the direct, or d, axis; and the quadrature or q, axis; that is, with the q-axis being at an angle of 90 degrees from the direct axis.

Shown above is the dqo transform as applied to the stator of a synchronous machine. There are three windings separated by 120 physical degrees. The three phase currents are equal in magnitude and are separated from one another by 120 electrical degrees. The three phase currents lag their corresponding phase voltages by . The d-q axis is shown rotating with angular velocity equal to , the same angular velocity as the phase voltages and currents. The d axis makes an angle with the A winding which has been chosen as the reference. The currents and are constant DC quantities.

### **Power for equivalence**

The power of a test estimates how likely it is that the test rejects the null hypothesis if the null hypothesis is indeed false. Because the null hypothesis for an equivalence test is often the opposite from the null hypothesis of a standard t-test of the population means, its power is expressed differently.

In equivalence testing, power is the likelihood that you will conclude that the population difference (or ratio) is within your equivalence limits when it actually is. If your test has low power, you may mistakenly conclude that you cannot claim equivalence when the difference (or ratio) is actually within the equivalence limits.

The following factors affect the power of your test.

#### **Sample size**

Larger samples give your test more power.

#### **Difference**

When the difference is close to the center of the two equivalence limits, your test has more power.

#### **Standard deviation**

Lower variability gives your test more power.

#### **Alpha**

Higher values for  $\alpha$  give your test more power. However,  $\alpha$  represents the probability of type I error. So increasing  $\alpha$  increases your chance of claiming equivalence when it is not true.

Use Power and Sample Size for 1-Sample Equivalence Test to examine the relationship between power, sample size, and difference when you want to evaluate the equivalence between the mean of a product or process and a target value.

Use these calculations for the following reasons:

- Before you collect data for a 1-sample equivalence test, to ensure that your design has adequate sample size to achieve acceptable power
  - After a 1-sample equivalence test, to improve the design for the next study
- For example, a quality analyst wants to determine whether the mean force required to open snack bag seals is within 10% of the target value of 4.2 N (Newtons). Before collecting the data for a 1-sample equivalence test, the analyst uses a power and sample size calculation to determine how large the sample must be to obtain a power of 80% (0.8).

Use Power and Sample Size for 2-Sample Equivalence Test to examine the relationship between power, sample size, and difference when you want to evaluate the equivalence between the test mean and the reference mean of independent samples.

Use these calculations for the following reasons:

- Before you collect data for a 2-sample equivalence test, to ensure that your design has an adequate sample size to achieve acceptable power
  - After a 2-sample equivalence test, to improve the design for the next study
- For example, a quality analyst wants to determine whether the mean amount of active ingredient in a generic brand of pain reliever is within 1 mg of the mean amount in a popular brand of pain reliever. Before collecting the data for a 2-sample equivalence test, the analyst uses a power and sample size calculation to determine how large the sample size must be to obtain a power of 90% (0.9).

### **How are dependent and independent samples different?**

Dependent samples are paired measurements for one set of items. Independent samples are measurements made on two different sets of items.

When you conduct a hypothesis test using two random samples, you must choose the type of test based on whether the samples are dependent or independent. Therefore, it's important to know whether your samples are dependent or independent:

- If the values in one sample affect the values in the other sample, then the samples are dependent.
- If the values in one sample reveal no information about those of the other sample, then the samples are independent.

Example of collecting dependent samples and independent samples

Consider a drug company that wants to test the effectiveness of a new drug in reducing blood pressure. They could collect data in two ways:

- Sample the blood pressures of the same people before and after they receive a dose. The two samples are dependent because they are taken from the same people. The people with the highest blood pressure in the first sample will likely have the highest blood pressure in the second sample.
- Give one group of people an active drug and give a different group of people an inactive placebo, then compare the blood pressures between the groups. These two samples would likely be independent because the measurements are from different people. Knowing something about the distribution of values in the first sample doesn't inform you about the distribution of values in the second.

Equivalence Test with Paired Data

Use Power and Sample Size for Equivalence Test with Paired Data to examine the relationship between power, sample size, and difference when you want to evaluate the equivalence between a test mean and a reference mean using paired observations.

Use these calculations for the following reasons:

- Before you collect data for an equivalence test with paired data, to ensure that your design has an adequate sample size to achieve acceptable power
- After an equivalence test with paired data, to improve the design for the next study

The paired equivalence test is useful for analyzing a set of dependent observations, such as the same set of items that were measured under two different conditions, or before-and-after measurements of the same person.

For example, an engineer for a vision care company wants to determine whether a new cleaning solution for contact lenses is as effective as the leading brand. The engineer recruits 14 participants for the study. Each participant will use a different cleaning solution on the contact lens in each eye. The engineer will measure the residual film on each contact lens of each person, which will create paired observations. Before the engineer collects the data for the equivalence test with paired data, she uses a power and sample size calculation to determine whether a sample size of 14 provides adequate power for the test.

### **Electromagnetic torque**

Milling is a machining process in which the piece material removal takes place intermittently. The milling is accomplished by the combination of two movements performed simultaneously. One of the movements is rotation of the cutter around its axis. The other is the movement of the milling machine table, which is attached to the piece to be milled. Thus, cutting parameters such as feedrate, cutting speed and cutting depth should be analyzed (Alves, 2002). According to Mao-Yue et al. (2009), parameters of the machining process should be monitored and adjusted automatically, which can improve the production quality and reduce the machining time. With advancements in applications using induction motors, the use of these motors on start and controlling of positioning systems are majority in the industry. An example of positioner machine is the coordinate table, which has applicability to devices milling, turning, drilling, and welding, among others.

The milling operation is one of the most important in the process of mechanical machining. The three-phase induction motor is widely used in various applications in industry, due to its relative simplicity of construction, maintenance, both operational and market low cost, as well as, the capacity to operate with a wide range of loads in adverse conditions. One of the factors that provided the advancement in driving of three-phase induction motor was the development of control strategies, specifically vector control strategies. In induction machine, the implementation of vector control for field orientation can be done by direct or indirect method. The direct field orientation enables the position of the flux is determined by measuring terminal quantities of the stator (voltages and currents). The advantage of using the stator terminal quantities lies in the fact to be sensitive only to the stator resistance (Altuna, 1997). In the development of controllers and estimators, and the use of vector control, the application of intelligent control techniques, such as fuzzy logic and artificial neural network (ANN), provided greater robustness to systems powered by three-phase induction motors. Fuzzy logic is a technique that allows the implementation of human experience in systems, considering the uncertainty of information, the ambiguities, for the development of controllers. The fuzzy inference model type TakagiSugeno (TS) is capable of representing, approximately or exactly, any nonlinear dynamics as combination of locally valid linear models, interpolating smoothly (Mozelli, 2008).

### **Torque equation**

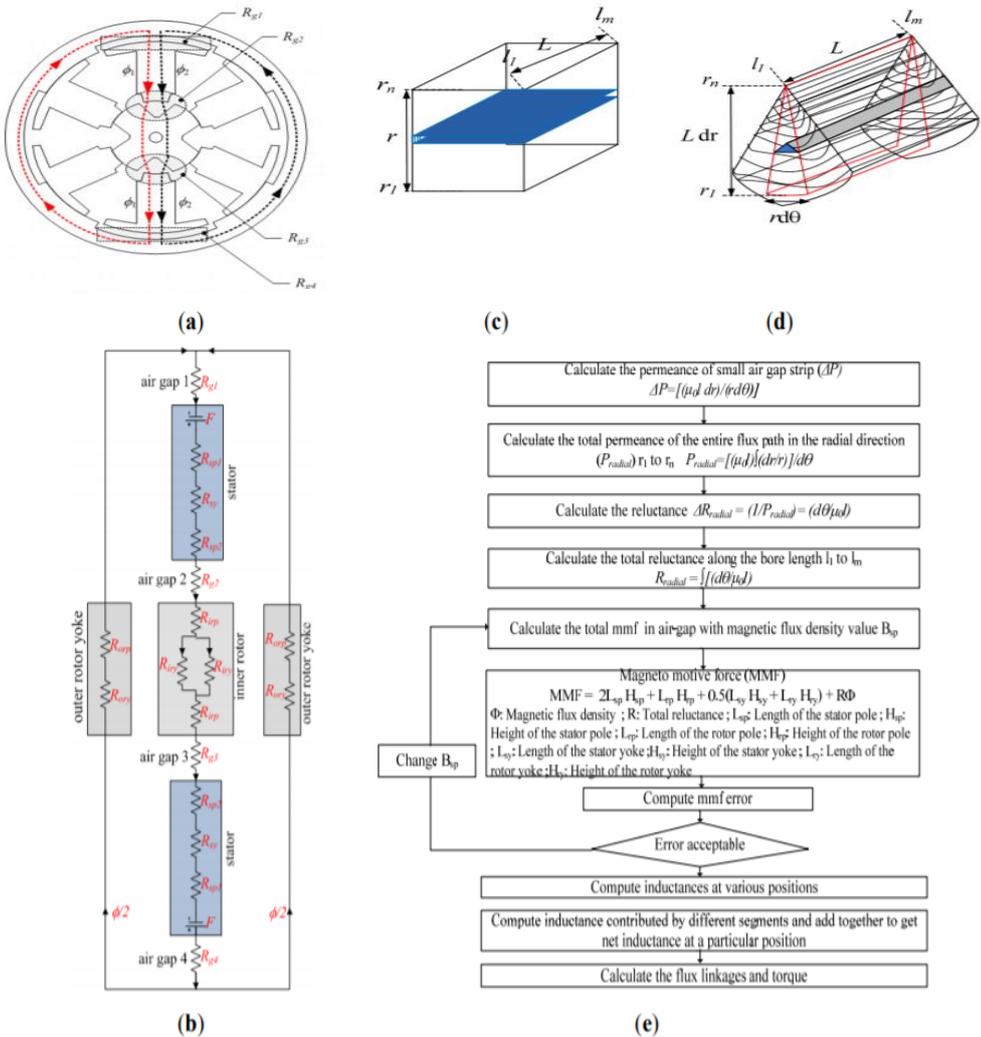
Whenever armature carries current in presence of flux, conductor experiences force which gives rise to the electromagnetic torque. In this section we shall derive an expression for the electromagnetic torque  $T_e$  developed in

a d.c machine. Obviously  $T_e$  will be developed both in motor and generator mode of operation. It may be noted that the direction of conductor currents reverses as we move from one pole to the other. This ensures unidirectional torque to be produced. The derivation of the torque expression is shown below.

$$\begin{aligned} \text{Let, } I_a &= \text{Armature current} \\ \text{Average flux density } B_{av} &= \frac{\bar{\phi} p}{\pi DL} \\ \text{Then, } \frac{I_a}{a} &= \text{Current flowing through each conductor.} \\ \text{Force on a single conductor} &= B_{av} \frac{I_a}{a} L \\ \text{Torque on a single conductor} &= B_{av} \frac{I_a}{a} L \frac{D}{2} \\ \text{Total electromagnetic torque developed, } T_e &= z B_{av} \frac{I_a}{a} L \frac{D}{2} \\ \text{Putting the value of } B_{av}, \text{ we get } T_e &= \frac{p z}{2\pi a} \phi I_a \end{aligned} \tag{37.2}$$

Thus we see that the above equation is once again applicable both for motor and generator mode of operation. The direction of the electromagnetic torque,  $T_e$  will be along the direction of rotation in case of motor operation and opposite to the direction of rotation in case of generator operation. When the machine runs steadily at a constant rpm then  $T_e = T_{load}$  and  $T_e = T_{pm}$ , respectively for motor and generator mode. The emf and torque equations are extremely useful and should be remembered by heart.

Magnetic circuit network: (a) Flux flow; (b) Reluctance network; (c) Iron core calculations; (d) Airgap calculations; (e) Algorithm used for calculations.



As the stator is excited with the power circuit the flux flow from the stator to the inner rotor yoke through air gap 2 (Rg2), enters the stator in the other face through air gap 3 (Rg3) in the bottom stator pole, moves to the outer yoke through air gap 4 (Rg4) and returns to the excited coil through the airgap1 (Rg1) in the upper stator pole from the outer rotor surface. The machine has two air gaps at the interaction teeth of the outer rotor-stator surface and the inner rotor-stator surface on both sides of the flux flow (Rg1–Rg4). The magnetic circuit considered is long flux

flow and hence the reluctance□

includes two reluctance circuits  $(R_{g1} + R_{g2} + R_{sp1} + R_{sy} + R_{sp2} + R_{sp1} + R_{irp} + 0.5R_{iry})$  on the upper surface and  $(R_{g3} + R_{g4} + R_g + R_{sp1} + R_{sy} + R_{sp2} + R_{sp1} + R_{irp})$  on the lower surface.

With  $R_{g1} = R_{g3}$  and  $R_{g2} = R_{g4}$  the net reluctance is computed as:

$$R_t = R_{inner} + \left(\frac{R_{ory} + R_{orp}}{2}\right) \quad (26)$$

$$R_{inner} = 2(R_{g1} + R_{g2} + R_{sp1} + R_{sy} + R_{sp2} + R_{sp1} + R_{irp}) + \left[\frac{(R_{iry})}{2}\right] \quad (27)$$

The flux tube path is chosen based on the position of the rotor with respect to the stator. Two flux tube paths are assumed under unaligned and aligned position with the flux entering the upper part leaving the lower tube exactly on the same side. At any partial aligned computation position the flux tube is assumed as three different paths with flux lines flowing uniformly in the center tube and in the other two tubes the flux entering the upper surface leaves on the opposite side of the lower surface. The surface area of flux linkage of the tubes depends on the rotor position with respect to that of the stator surface at any time instance. Within the flux tube for each of the parts of the machine such as the stator, rotor cores, air-gap etc., a small strip is integrated along the longitudinal axis and then integrated over the surface under considerations from  $l1$  to  $l_m$  then from  $r1$  to  $r_n$  as in Figure 4c and Figure 4d for the iron core and the air gap respectively. Figure 4e shows the algorithm used in the computation of the flux tube analysis.

### UNIT III - SMALL SIGNAL MODELING

- A small signal model is a linear model which is independent of amplitude. It may or may not have time dependence (i.e. capacitors).
- The small signal model for a nonlinear component such as a BJT is a linear model about some nominal operating point. The deviations from the operating point are small enough that it approximates the nonlinear component over a limited range of amplitudes.

Illustration of the *pn* diode:

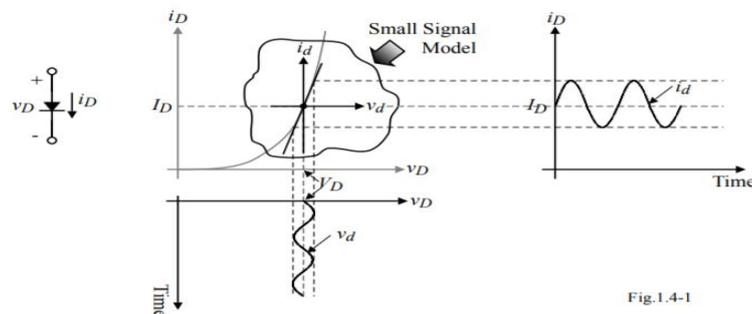


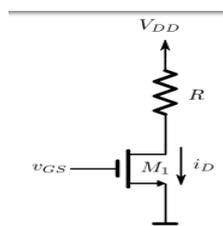
Fig.1.4-1

Small-signal modeling is a common analysis technique in electronics engineering which is used to approximate the behavior of electronic circuits containing nonlinear devices with linear equations. It is applicable to electronic circuits in which the AC signals, the time-varying currents and voltages in the circuit, have a small magnitude compared to the DC bias currents and voltages. A small-signal model is an AC equivalent circuit in which the nonlinear circuit elements are replaced by linear elements whose values are given by the first-order (linear) approximation of their characteristic curve near the bias point.

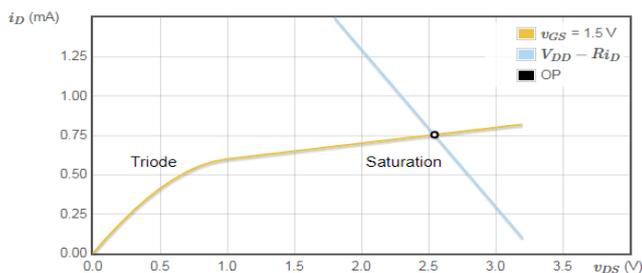
The CMOS transistor is normally used as an amplifier when it is working in the saturation region (it has a low dependence with  $V_{DS}$  and a high sensitivity to  $V_{GS}$ ). To characterize the transistor for that purpose, it is used a model that considers that small signals are injected in the terminals of the transistor. That model linearizes the  $I_D$ - $V_{GS}$  and  $I_D$ - $V_{DS}$  curves around a point of operation, called the bias or operating point.

#### **The bias or operating point**

Consider the following circuit:



A gate voltage  $v_{GS}$  is applied to the transistor. A current will be generated that will match the current that flows through the resistance. This will in turn generate a voltage drop across the resistance and set the drain-source voltage of the transistor. This will lead to a stable condition for the transistor, for which we call bias or operating point.



The slider represents  $R$ . As you decrease  $R$  (go left), the current sunk by the transistor has a small voltage drop at the resistance and the operating point goes to higher  $v_{DS}$ . If you increase  $R$  (go right), the voltage drop is higher and  $v_{DS}$  decreases. If the resistance was too large, it would put the transistor in triode region for this  $v_{GS}$ .

$R = 1k\Omega$

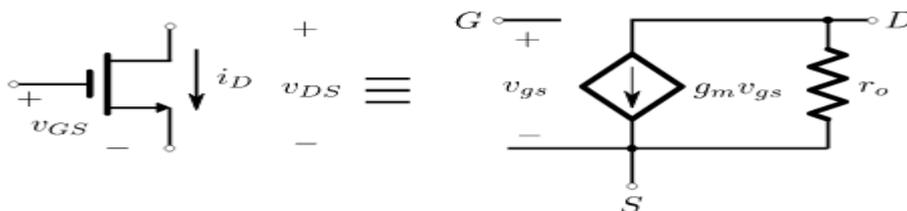
### Linearizing the curves

If a small signal  $v_{gs}$  is superimposed on a DC signal  $V_{GS}$  to produce  $v_{GS} = V_{GS} + v_{gs}$  at the gate of the transistor, the DC signal sets the bias and the small-signal will swing around that bias point. The curves around the bias point are not linear and to get a simple model for small signals that can be analyzed, we approximate the curves to lines to get a linear model.

The linear model is based on taking the derivatives of the drain current with respect to the voltages at the terminals of the transistor.

### The small-signal equivalent model

First let us find what components should make up this model. First we see that the transistor is a current source controlled by the gate voltage, so a voltage-controlled current source should be there. Next, we see that the current is also influenced by the drain-source voltage from the channel length modulation, which translates into an output resistance. There is no input resistance because there is no current flowing through the gate.



Now it is time to determine the values of the transconductance  $g_m$  and the output resistance  $r_o$ .

Transconductance  $g_m$

$g_m$  is the change in current caused by a change in  $v_{GS}$ , so it is:

$$g_m = \partial i_D / \partial v_{GS}$$

The equation for  $g_m$  depends whether the transistor is in triode or in saturation.

$g_m$ in triode	$g_m$ in saturation
$\frac{\partial \frac{1}{2} \mu C_{ox} \frac{W}{L} (2(v_{GS} - V_{TH})v_{DS} - v_{DS}^2)}{\partial v_{GS}}$ $\mu C_{ox} \frac{W}{L} v_{DS}$	$\frac{\partial \frac{1}{2} \mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH})^2}{\partial v_{GS}}$ $\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$

When in triode,  $g_m$  does not really depend on  $v_{GS}$  because  $i_D$  is linear with  $v_{GS}$  in this region. However,  $v_{DS}$  affects the transconductance linearly. When in saturation,  $g_m$  depends on  $V_{GS}$  and it is a parameter that can be tweaked to control the gain of the transistor.

### Output resistance $r_o$

$r_o$  is the inverse of a change in current caused by a change in the drain-source voltage, so it is:

$$1/r_o = g_o = \partial i_D / \partial v_{DS}$$

$g_o$  is the output conductance and is commonly used in exchange of  $r_o$  when it simplifies the equations. The equation for  $g_o$  depends whether the transistor is in triode or in saturation.

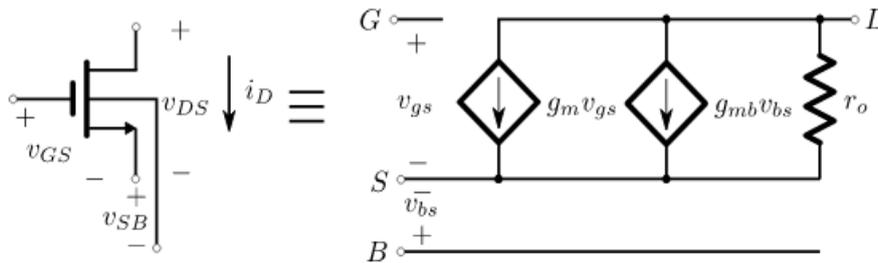
$g_o$ in triode	$g_o$ in saturation
$\frac{\partial \frac{1}{2} \mu C_{ox} \frac{W}{L} (2(v_{GS} - V_{TH})v_{DS} - v_{DS}^2)}{\partial v_{DS}}$	$\frac{\partial \frac{1}{2} \mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH})^2 (1 + \lambda v_{DS})}{\partial v_{GS}}$
$\frac{1}{2} \mu C_{ox} \frac{W}{L} (2(v_{GS} - V_{TH}) - 2v_{DS})$	$\frac{1}{2} \mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH})^2 \lambda$
$\mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH} - v_{DS})$	$\approx I_D \lambda$

The output resistance is influenced both by  $v_{GS}$  and  $v_{DS}$  when the transistor is in triode. When in saturation, the output resistance is only defined by the bias current.

From here we see that the transistor is not a very good amplifier in the triode region, because its gain is changed with the output voltage and we can only play with the transistor's size to control the gain. It is not a very good current source either, because its gain (and therefore its current) and its output resistance changes with the output voltage. The transistor is usually used in the saturation region for amplification. Also, to reduce the distortion caused by the amplification, the bias point should be somewhere in the middle of the saturation region, so that large swings of the drain voltage will not bring the transistor into the triode region or saturate to the power supply voltage.

### Include body effect

The body effect changes the threshold voltage, which in turn affects the drain current. How can this be incorporated in the model? The body is like another gate (it is sometimes called a back-gate) and it is modelled as just that:



Since  $v_{BS}$  is always negative, the current direction of the  $g_{mb}$  current source is actually the opposite of the  $g_m$  current source. To calculate the value of  $g_{mb}$ , we have to realize that there is a cascade of effects ( $v_{BS}$  to  $V_{TH}$  and  $V_{TH}$  to  $i_D$ ):

$$g_{mb} = \frac{\partial i_D}{\partial V_{TH}} \frac{\partial V_{TH}}{\partial v_{BS}}$$

$\frac{\partial V_{TH}}{\partial v_{BS}}$  is expressed as:

$$\frac{\partial V_{TH}}{\partial v_{BS}} = - \frac{\partial V_{T0n} + \gamma (\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f})}{\partial v_{BS}}$$

$$\frac{\partial V_{TH}}{\partial v_{BS}} = - \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$$

As in the other cases, the expression depends if the transistor is in triode or in saturation

$g_{mb}$ in triode	$g_{mb}$ in saturation
$g_{mb} = \frac{\partial i_D}{\partial V_{TH}} \frac{\partial V_{TH}}{\partial v_{BS}}$ $g_{mb} = \frac{\partial \frac{1}{2} \mu C_{ox} \frac{W}{L} (2(v_{GS} - V_{TH})v_{DS} - v_{DS}^2)}{\partial V_{TH}} \frac{\partial V_{TH}}{\partial v_{BS}}$ $\frac{1}{2} \mu C_{ox} \frac{W}{L} (-2v_{DS}) \frac{\partial V_{TH}}{\partial v_{BS}}$ $-\mu C_{ox} \frac{W}{L} v_{DS} \frac{\partial V_{TH}}{\partial v_{BS}}$ $-g_m \frac{\partial V_{TH}}{\partial v_{BS}}$ $g_m \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$	$\frac{\partial \frac{1}{2} \mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH})^2}{\partial V_{TH}} \frac{\partial V_{TH}}{\partial v_{BS}}$ $\frac{\partial \mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH})^2}{\partial V_{TH}} \frac{\partial V_{TH}}{\partial v_{BS}}$ $-\mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH}) \frac{\partial V_{TH}}{\partial v_{BS}}$ $-g_m \frac{\partial V_{TH}}{\partial v_{BS}}$ $g_m \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$

The model has three parameters: the gate-source transconductance  $g_m$ , the bulk-source transconductance  $g_{mb}$  and the output resistance  $r_o$ . The expressions for these parameters are summarized in the following table:

Parameter	Triode	Saturation
$g_m$	$\mu C_{ox} \frac{W}{L} v_{DS}$	$\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$
$r_o$	$\frac{1}{\mu C_{ox} \frac{W}{L} (v_{GS} - V_{TH} - v_{DS})}$	$\frac{1}{I_D \lambda}$
$g_{mb}$	$g_m \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$	

Wind power is one of the most promising renewable energy sources after the progress undergone during the last decade. However, its integration into power systems has a number of technical challenges concerning security of supply, in-term of reliability, availability and power quality. An unbalance of voltage may occur on long distance power transmission lines as shown in figure 1. It occurs not infrequently in the case of generating stations supplying mixed polyphase and single phase loads. The predetermination of the performance of three-phase induction motors under such conditions is a matter of importance that there is a reduction in the maximum load which an induction motor is capable of carrying safely when supplied with unbalanced voltages, but the extent of this reduction has not received the attention which it merits. It is carried out experiments on the performance of induction machines on unbalanced voltages, but they have assumed that the load limit of the motor is reached when the current in one phase reaches its full load value, and hence their experiments underrate the performance of the motor [2]. It is treated the matter analytically and derived certain simple formulae to predetermine the output, but his results have not received conclusive experimental verification. The various conditions of symmetrical components, unsymmetrical components, balance and unbalance conditions, positive, negative, zero sequence and harmonic pulsations, voltage unbalance theory and factor have been discussed. The output of the simulation result has been verified by experimental calculation. Graphical method of resolving an unsymmetrical voltage system into two symmetrical systems is also described.

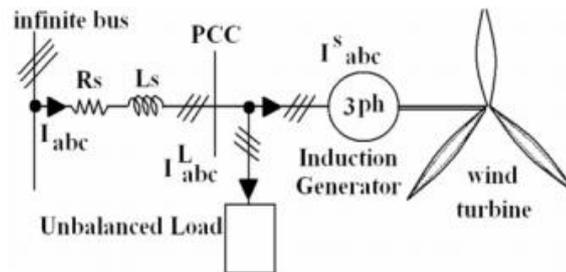


Figure 1: Physical Connection in the Power System

### Steady State Operation of Three-Phase Induction Motor with Asymmetrical External Stator Impedances and Voltage Asymmetry

In the following section a three-phase induction motor will be supposed. Due to stator asymmetry, the existing negative-sequence mmf will produce a negative-sequence stator current of  $(2 - s)$  frequency (s is the slip, and  $\omega_s$  is the stator frequency) as the slip of the rotor compared to the negative sequence mmf is  $(2 - s)$ . Figure 1 shows the negative and positive-sequence symmetrical-component impedances of the

machine; the same assumptions are made as in [10], however the rotor is symmetrical now, As the negative-sequence impedance is smaller than the impedance of the symmetrical machine, it can be seen that in case of relatively small asymmetries, negative sequence currents of large values can occur. The effect of asymmetry is the smallest in the case of standstill, and is the greatest around synchronous speed. Therefore, due to asymmetry, at least one of the stator phase currents is greater than for a symmetrical machine, which results in higher loads and heatings of the stator windings. Besides the increase of the stator current, other undesirable effects will also occur, such as the reduction of the mechanical power, as the motor tends to revolve in the backward direction due to the negative sequence mmf. As a consequence, the negative sequence torque -- which exists because of the asymmetry -- could reduce the positive sequence torque. Also the pulsating torques can cause undesirable effects, too. In deriving the total equivalent circuit for the steady state operation of the asymmetrical machine the method presented in [10] will be followed. Anyhow, for an asymmetrical machine two main groups of calculation methods can be utilized. The first group consists of those methods, where the phase-coordinate equations of the asymmetrical machine are directly solved. In case of the second group, the phase-coordinate equations are simplified by transformations -- and in some cases by partitions too -- and the simplified equations are solved. (Sometimes transformations are even used if they do not lead to simpler equations, but the physical phenomena become clearer.) If it is the purpose only to derive the results, a method of the first group must be used, e.g. in transient operation, the first order differential equations are solved by a digital computer. In this case the additive components due to asymmetry will be separately not present, although they can be calculated by using the conditions of the symmetrical machine too. In steady-state operation, first the voltage equations must be solved for the currents if a method of the first group is used. This can be also accomplished by the direct inversion of the impedance matrix; however, if partitioning is also possible, it is useful to carry out this simplification, too. However, if the additive components are necessary to be in an analytical form, two methods can be followed. In case of partially asymmetrical machines, dyadic methods can be used; or by using the initial conditions of symmetrical and asymmetrical machine, the differential component will be equal to the additive one. In the followings a method of the second group will be used as the physical phenomena become clear this way. The external impedances of the stator are  $Z_a$ ,  $Z_b$ , and  $Z_c$  respectively, the stator is star connected. The symmetrical components of the stator voltages are  $U_{s0}$ ,  $U_{s1}$ , and  $U_{s2}$ , and the symmetrical component equation is

$$\begin{bmatrix} U_{s0} \\ U_{s1} \\ U_{s2} \end{bmatrix} = \begin{bmatrix} Z_0 & Z_1 \\ Z_1 & Z_2 \\ Z_2 & Z_0 \end{bmatrix} \begin{bmatrix} I_{s0} \\ I_{s1} \\ I_{s2} \end{bmatrix},$$

### Single-Phase Motors

Single phase motors are the most familiar of all electric motors because they are extensively used in home appliances, shops, offices etc. It is true that single phase motors are less efficient substitute for 3-phase motors but 3-phase power is normally not available except in large commercial and industrial establishments. Since electric power was originally generated and distributed for lighting only, millions of homes were given single-phase supply. This led to the development of single-phase motors. Even where 3-phase mains are present, the single-phase supply may be obtained by using one of the three lines and the neutral. Single-phase induction motors are usually two-pole or four-pole, rated at 2 hp or less, while slower and larger motor can be manufactured for special purposes. They are widely used in domestic appliances and for a very large number of low power drives in industry. The single phase induction motor resembles, three-phase, squirrel-cage motor except that, single phase induction motor has no starting torque and some special arrangement have to be made to make it as self starting. In this chapter, we shall focus our attention on the construction, working and characteristics of commonly used single-phase motors.

### Types of Single-Phase Motors:

**Single-phase motors are generally built in the fractional-horsepower range and may be classified into the following four basic types:**

#### 1. Single-phase induction motors

- (i) split-phase type
- (ii) capacitor start type
- (iii) capacitor start capacitor run type
- (v) shaded-pole type

#### 2. A.C. series motor or universal motor

#### 3. Repulsion motors

- (i) Repulsion-start induction-run motor
- (ii) Repulsion-induction motor

#### 4. Synchronous motors

(i) Reluctance motor

(ii) Hysteresis motor

#### Single-Phase Induction Motors

A single phase induction motor is very similar to a 3-phase squirrel cage induction motor. Unlike a 3-phase induction motor, a single-phase induction motor is not self starting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though singlephase current is flowing through the stator winding. This method of starting is generally not convenient for large motors. Figure 7.2 shows picture of single phase induction motor.



Figure 7.1 Single phase induction motor.

#### Construction of single phase induction motor

The construction parts on of single phase induction motor consist of main two parts: stationary stator and revolving rotor. The stator separate from rotor by small air gap have ranges from 0.4 mm to 4 mm depends to size of motor.

#### Stator

The single-phase motor stator has a laminated iron core with two windings arranged perpendicularly, One is the main and the other is the auxiliary winding or starting winding as showing in the figure 7.2. It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations.



Figure 7.2 Stator of single phase induction motor.

#### Rotor

The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

#### (i) Squirrel cage rotor:

It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. 7.3]. This forms a permanently shortcircuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator. Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of single phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque. In this type of rotor the bars conductor are skew to reduce the noise.



Figure 7.3 Squirrel cage rotor.

### Wound rotor:

It consists of a laminated cylindrical core and carries a single phase winding, similar to the one on the stator. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The two brushes are connected to a single phase star-connected rheostat as shown in Figure 7.4. At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the two brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.



Figure 7.4 wound rotor of single phase induction motor.

### Principle Of Work:

A single-phase induction motor is not self starting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though single-phase current is flowing through the stator winding. This method of starting is generally not convenient for large motors. Figure 7.5 shows single-phase induction motor having a squirrel cage rotor and a single phase distributed stator winding. Such a motor inherently does not develop any starting torque and, therefore, will not start to rotate if the stator winding is connected to single-phase A.C. supply. However, if the rotor is started by auxiliary means, the motor will quickly attain the final speed. This strange behavior of single-phase induction motor can be explained on the basis of double-field revolving theory.

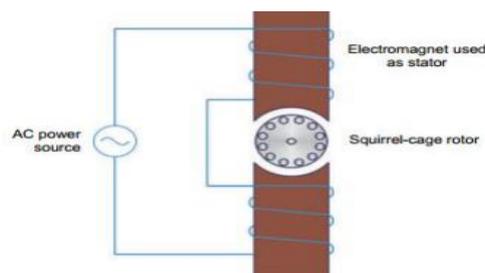


Figure 7.5 single-phase induction motor.

### Operation of Single phase induction motor

- (i) When stator winding is energized from a.c. supply, a rotating magnetic field (RMF) is set up which rotates round the stator at synchronous speed  $N_s (= 120 f/P)$ , when  $f =$  frequency and  $P$  No. of poles .
- (ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary . Due to the relative speed between the rotating flux and the stationary rotor, electrical motive force (EMF) are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors (Figure 7.6). The flux from the stator will cut the coil in the rotor and since the rotor coils are short circuited, according to Faraday's law of electromagnetic induction, current will start flowing in the coil of the rotor.

- (iii) (iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field with speed  $N = N_s (1 - S)$  when  $S =$  slip and  $N =$  rotor speed (Figure 7.6).

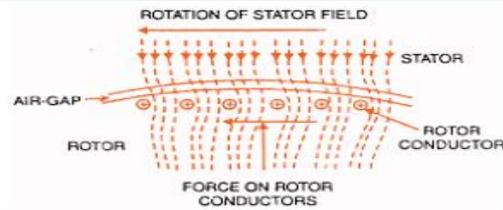


Figure 7.6 Transmission of Rotating magnetic field

**Cross-field theory:**

The principle of operation of a single-phase induction motor can be explained from the cross-field theory. As soon as the rotor begins to turn, a speed emf  $E$  is induced in the rotor conductors, as they cut the stator flux  $F_s$ . This voltage increases as the rotor speed increases. It causes current  $I_r$  to flow in the rotor bars facing the stator poles as shown in figure 7.7. These currents produce an ac flux  $F_R$  which act at right angle to the stator flux  $F_s$ . Equally important is the fact that  $F_R$  does not reach its maximum value at the same time as  $F_s$  does, in effect,  $F_R$  lags almost  $90^\circ$  behind  $F_s$ , owing to the inductance of the rotor. The combined action of  $F_s$  and  $F_R$  produces a revolving magnetic field, similar to that in a three-phase motor. The value of  $F_R$  increases with increasing speed, becoming almost equal to  $F_s$  at synchronous speed. The flux rotates counterclockwise in the same direction as the rotor and it rotates at synchronous speed irrespective of the actual speed of the rotor. As the motor approaches synchronous speed,  $F_R$  becomes almost equal to  $F_s$  and a nearly perfect revolving field is produced.

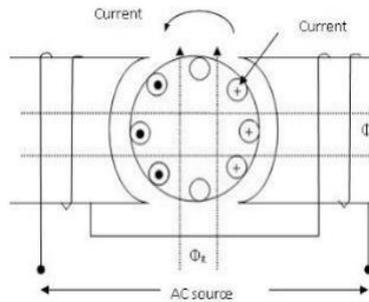


Figure 7.7 Current induced in the rotor bars due to rotation.

**Double-field revolving theory:**

When the stator winding (distributed one as stated earlier) carries a sinusoidal current (being fed from a single-phase supply), a sinusoidal space distributed mmf, whose peak or maximum value pulsates (alternates) with time, is produced in the air gap. This sinusoidal varying flux ( $\phi$ ) is the sum of two rotating fluxes or fields, the magnitude of which is equal to half the value of the alternating flux ( $\phi / 2$ ), and both the fluxes rotating synchronously at the speed, in opposite directions. The first set of figures (Figure 7.8a (i-iv)) show the resultant sum of the two rotating fluxes or fields, as the time axis (angle) is changing from  $\theta = 0^\circ$  to  $\pi^\circ (180^\circ)$ . Figure 7.8b shows the alternating or pulsating flux (resultant) varying with time or angle.

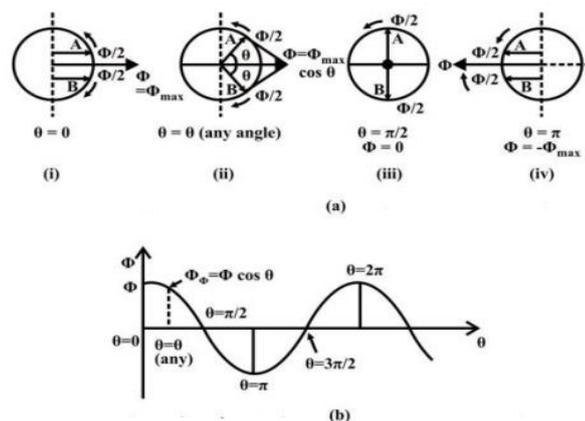


Figure 7.8 Double field revolving.

The flux or field rotating at synchronous speed, say, in the anticlockwise direction, i.e. the same direction, as that of the motor (rotor) taken as positive induces EMF (voltage) in the rotor conductors. The rotor is a squirrel cage one, with bars short circuited via end rings. The current flows in the rotor conductors, and the electromagnetic torque is produced in the same direction as given above, which is termed as positive (+ve). The other part of flux or field rotates at the same speed in the opposite (clockwise) direction, taken as negative. So, the torque produced by this field is negative (-ve), as it is in the clockwise direction, same as that of the direction of rotation of this field. Two torques are in the opposite direction, and the resultant (total) torque is the difference of the two torques produced. Let the flux  $\phi_1$  rotate in anti clockwise direction and flux  $\phi_2$  in clockwise direction. The flux  $\phi_1$  will result in the production of torque  $T_1$  in the anti clockwise direction and flux  $\phi_2$  will result in the production of torque  $T_2$  in the clockwise direction. Thus the point of zero slip for one field corresponds to 200% slip for the other as explained later. The value of 100% slip (standstill condition) is the same for both the fields. This fact is illustrated in Figure 7.9. At standstill, these two torques are equal and opposite and the net torque developed is zero. Therefore, single-phase induction motor is not self-starting. Note that each rotating field tends to drive the rotor in the direction in which the field rotates.

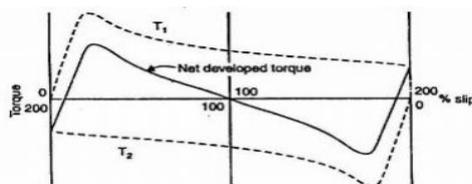


Figure 7.9 Speed Torque characteristics.

Now assume that the rotor is started by spinning the rotor or by using auxiliary circuit, in say clockwise direction.

The flux rotating in the clockwise direction is the forward rotating flux ( $\phi_f$ ) and that in the other direction is the backward rotating flux ( $\phi_b$ ). The slip w.r.t. the forward flux will be

$$S_f = \frac{N_s - N}{N_s} = 1 - \frac{N}{N_s} \text{ or } \frac{N}{N_s} = 1 - S$$

The rotor rotates opposite to the rotation of the backward flux. Therefore, the slip w.r.t. the backward flux will be

$$S_b = \frac{N_s - (-N)}{N_s} = \frac{N_s + N}{N_s} = 1 + \frac{N}{N_s} = 1 + (1 - S) = 2 - S$$

Thus for forward rotating flux, slip is  $s$  (less than unity) and for backward rotating flux, the slip is  $2 - s$  (greater than unity). Since for usual rotor resistance/reactance ratios, the torques at slips of less than unity are greater than those at slips of more than unity, the resultant torque will be in the direction of the rotation of the forward flux. Thus if the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor.

### Making Single-Phase Induction Motor Self-Starting:

The single-phase induction motor is not self starting and it is undesirable to resort to mechanical spinning of the shaft or pulling a belt to start it. To make a single-phase induction motor self-starting, we should somehow produce a revolving stator magnetic field. This may be achieved by converting a single-phase supply into two-phase supply through the use of an additional winding. When the motor attains sufficient speed, the starting means (i.e., additional winding) may be removed depending upon the type of the motor. As a matter of fact, single-phase induction motors are classified and named according to the method employed to make them self-starting. (i) Split-phase motors-started by two phase motor action through the use of an auxiliary or starting winding. (ii) Capacitor start motors-started by two-phase motor action through the use of an auxiliary winding and a capacitor. (iii) Capacitor start Capacitor run motors-started by two-phase motor action through the use of an auxiliary winding and two capacitors. (v) Shaded-pole motors-started by the motion of the magnetic field produced by means of a shading coil around a portion of the pole structure.

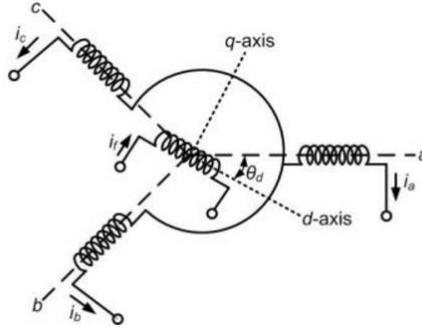
### Split-phase induction motors

The stator of a split-phase induction motor is provided with an auxiliary or starting winding  $S$  in addition to the main or running winding  $M$ . The starting winding is located  $90^\circ$  electrical from the main winding and the picture of split phase induction motor [See Fig.7.12 (i)] and operates only during the brief period when the motor starts up. The two windings are so designed that the starting winding  $S$  has a high resistance and relatively small reactance while the main winding  $M$  has relatively low resistance and large reactance to be as inductance (the current delay with voltage) to make shifting current as shown in the schematic connections in Figure 7.12 (ii). Consequently, the currents flowing in the two windings have reasonable phase difference  $c$  ( $25^\circ$  to  $30^\circ$ ) as shown in the phasor diagram this shifting in current is necessary for starting torque in Figure 7.12 (iii). Figure 7.12 (iv) shows typical torque speed characteristics.

## UNIT IV - MODELING OF SYNCHRONOUS MACHINE

### SYNCHRONOUS MACHINE INDUCTANCES:

The schematic diagram of a synchronous generator is shown in Fig. 1.15. This contains three stator windings that are spatially distributed. It is assumed that the windings are wye-connected. The winding currents are denoted by  $i_a$ ,  $i_b$  and  $i_c$ . The rotor contains the field winding the current through which is denoted by  $i_f$ . The field winding is aligned with the so-called direct ( $d$ ) axis. We also define a quadrature ( $q$ ) axis that leads the  $d$ -axis by  $90^\circ$ . The angle between the  $d$ -axis and the  $a$ -phase of the stator winding is denoted by  $\theta_d$ .



**Fig. 1.15 Schematic diagram of a synchronous generator.**

Let the self-inductance of the stator windings be denoted by  $L_{aa}$ ,  $L_{bb}$ ,  $L_{cc}$  such that

$$L_s = L_{aa} = L_{bb} = L_{cc} \quad (1.80)$$

and the mutual inductance between the windings be denoted as

$$-M_s = L_{ab} = L_{bc} = L_{ca} \quad (1.81)$$

The mutual inductances between the field coil and the stator windings vary as a function of  $\theta_d$  and are given by

$$L_{af} = M_f \cos \theta_d \quad (1.82)$$

$$L_{bf} = M_f \cos(\theta_d - 120^\circ) \quad (1.83)$$

$$L_{cf} = M_f \cos(\theta_d + 120^\circ) \quad (1.84)$$

The self-inductance of the field coil is denoted by  $L_{ff}$ .

The flux linkage equations are then given by

$$\lambda_a = L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f = L_s i_a - M_s (i_b + i_c) + L_{af}i_f \quad (1.85)$$

$$\lambda_b = L_s i_b - M_s (i_a + i_c) + L_{bf}i_f \quad (1.86)$$

$$\lambda_c = L_s i_c - M_s (i_a + i_b) + L_{cf}i_f \quad (1.87)$$

For balanced operation we have

$$\lambda_f = L_{af}i_a + L_{bf}i_b + L_{cf}i_c + L_{ff}i_f \quad (1.88)$$

$$i_a + i_b + i_c = 0$$

Hence the flux linkage equations for the stator windings (1.85) to (1.87) can be modified as

$$\lambda_a = (L_s + M_s)i_a + L_{af}i_f \quad (1.89)$$

$$\lambda_b = (L_s + M_s)i_b + L_{bf}i_f \quad (1.90)$$

$$\lambda_c = (L_s + M_s)i_c + L_{cf}i_f \quad (1.91)$$

For steady state operation we can assume

$$i_f = I_f = \text{constant} \quad (1.92)$$

Also assuming that the rotor rotates at synchronous speed  $\omega_s$  we obtain the following two equations

$$\omega_s = \frac{d\theta_d}{dt} \quad (1.93)$$

$$\theta_d = \omega_s t + \theta_{d0} \quad (1.94)$$

where  $\theta_{d0}$  is the initial position of the field winding with respect to the phase- $a$  of the stator winding at time  $t = 0$ . The mutual inductance of the field winding with all the three stator windings will vary as a function of  $\theta_d$ , i.e.,

$$L_{af} = M_f \cos(\omega_s t + \theta_{d0}) \quad (1.95)$$

$$L_{bf} = M_f \cos(\omega_s t + \theta_{d0} - 120^\circ) \quad (1.96)$$

$$L_{cf} = M_f \cos(\omega_s t + \theta_{d0} + 120^\circ) \quad (1.97)$$

Substituting (1.92), (1.94), (1.95), (1.96) and (1.97) in (1.89) to (1.91) we get

Substituting (1.92), (1.94), (1.95), (1.96) and (1.97) in (1.89) to (1.91) we get

$$\lambda_a = (L_s + M_s) i_a + M_f I_f \cos(\omega_s t + \theta_{d0}) \quad (1.98)$$

$$\lambda_b = (L_s + M_s) i_b + M_f I_f \cos(\omega_s t + \theta_{d0} - 120^\circ) \quad (1.99)$$

$$\lambda_c = (L_s + M_s) i_c + M_f I_f \cos(\omega_s t + \theta_{d0} + 120^\circ) \quad (1.100)$$

Since we assume balanced operation, we need to treat only one phase. Let the armature resistance of the generator be  $R_a$ . The generator terminal voltage is given by

$$v_a = -R_a i_a - \frac{d\lambda_a}{dt} \quad (1.101)$$

where the negative sign is used for generating mode of operation in which the current leaves the terminal. Substituting (1.98) in (1.101) we get

$$v_a = -R_a i_a - (L_s + M_s) \frac{di_a}{dt} + M_f I_f \omega_s \sin(\omega_s t + \theta_{d0}) \quad (1.102)$$

The last term of (1.102) is the internal emf  $e_a$  that is given by

$$e_a = \sqrt{2} |E_i| \sin(\omega_s t + \theta_{d0}) \quad (1.103)$$

where the rms magnitude  $|E_i|$  is proportional to the field current

$$|E_i| = \frac{\omega_s M_f I_f}{\sqrt{2}} \quad (1.104)$$

Since  $\theta_{d0}$  is the position of the  $d$ -axis at time  $t = 0$ , we define the position of the  $q$ -axis at that instant as

The last term of (1.102) is the internal emf  $e_a$  that is given by

$$e_a = \sqrt{2} |E_i| \sin(\omega_s t + \theta_{d0}) \quad (1.103)$$

where the rms magnitude  $|E_i|$  is proportional to the field current

$$|E_i| = \frac{\omega_s M_f I_f}{\sqrt{2}} \quad (1.104)$$

Since  $\theta_{d0}$  is the position of the  $d$ -axis at time  $t = 0$ , we define the position of the  $q$ -axis at that instant as

$$\delta = \theta_{d0} - 90^\circ \quad (1.105)$$

Therefore (1.94) can be rewritten as

$$\theta_d = \omega_s t + \delta + 90^\circ \quad (1.106)$$

Substituting (1.105) in (1.103) we get

$$e_a = \sqrt{2} |E_i| \cos(\omega_s t + \delta) \quad (1.107)$$

Hence (1.102) can be written as

$$v_a = -R_a i_a - (L_s + M_s) \frac{di_a}{dt} + e_a \quad (1.108)$$

Contd..

## Torque Equation of an Induction Motor

The developed **Torque** or **Induced Torque Equation** in a machine is defined as the Torque generated by the electric to mechanical power conversion. The torque is also known as **Electromagnetic Torque**. This developed torque in the motor differs from the actual torque available at the terminals of the motor, which is almost equal to the friction and windage torques on the machine.

The developed torque equation is given as

$$T_d = \frac{\text{Mechanical power developed}}{\text{Mechanical angular velocity of the rotor}} = \frac{P_{md}}{\omega_r} \dots \dots (1)$$

Since,

$$P_{md} = (1 - s)P_g \quad \text{and} \quad \omega_r = (1 - s)\omega_s$$

$$T_d = \frac{(1 - s)P_g}{(1 - s)\omega_s} = \frac{P_g}{\omega_s} \dots \dots (2)$$

The above equation expresses the developed torque directly in terms of the air gap power  $P_g$  and the synchronous speed  $\omega_s$ . Since  $\omega_s$  is constant and independent of the load conditions. If the value of the  $P_g$  is known then, the developed torque can be found directly. The air gap power  $P_g$  is also called as the **Torque in Synchronous Watts**. Synchronous Watt is the torque that develops the power of 1 Watt when the machine is running at synchronous speed.

$$\text{Output power } P_o = \omega_r T_{load} \dots \dots (3)$$

$$T_{load} = \frac{P_o}{\omega_r} = \frac{P_{md} - P_{rot}}{\omega_r} \dots \dots (4)$$

Now, the electrical power generated in the rotor is given by the equation shown below.

$$\begin{aligned} \text{Electrical power} &= 3E_{2s}I_{2s}\cos\phi_{2s} \text{ Watt} \\ &= 3E_{2s} \times \frac{E_{2s}}{Z_{2s}} \times \frac{R_2}{Z_{2s}} = \frac{3 E_{2s}^2 R_2}{Z_{2s}^2} \\ &= \frac{3 s^2 E_{20}^2 R_2}{R_2^2 + (sX_{20})^2} \dots \dots \dots (5) \end{aligned}$$

These electrical powers are dissipated as  $I^2R$  losses or copper loss in the rotor circuit. Input power to the rotor is given as

$$\begin{aligned} \text{Input Power to the rotor} &= 2\pi n_s T_d \\ s \times \text{rotor input} &= \text{rotor copper loss} \end{aligned}$$

$$s \times 2\pi n_s T_d = \frac{3 s^2 E_{20}^2 R_2}{R_2^2 + (sX_{20})^2}$$

$$T_d = \frac{3 E_{20}^2}{2\pi n_s} \times \frac{s R_2}{R_2^2 + s^2 X_{20}^2} \dots \dots \dots (6)$$

$$T_d = \frac{k s E_{20}^2 R_2}{R_2^2 + s^2 X_{20}^2} \dots \dots \dots (7)$$

$$k = \frac{3}{2\pi n_s} = \frac{3}{\omega_s} = \text{constant} \dots \dots \dots (8)$$

**Starting Torque Of Induction Motor**

At the start condition the value of  $s = 1$ . Therefore, the starting is obtained by putting the value of  $s = 1$  in the equation (6), we get

$$T_{dst} = \frac{3 E_{20}^2 R_2}{2\pi n_s (R_2^2 + X_{20}^2)}$$

The starting torque is also known as **Standstill Torque**.

**Torque Equation at Synchronous Speed**

At synchronous speed,  $s = 0$  and hence developed torque  $T_d = 0$ . At synchronous speed, developed torque is zero.

$$\frac{E_{20}}{E_1} = \frac{T_{e2}}{T_{e1}} \dots \dots \dots (9)$$

$$E_{20} = \frac{T_{e2}}{T_{e1}} E_1$$

Therefore,

$$T_d = \frac{3}{2\pi n_s} \left( \frac{T_{e2}}{T_{e1}} \right)^2 E_1^2 \frac{s R_2}{R_2^2 + s^2 X_{20}^2} \dots \dots \dots (10)$$

Let,

$$\frac{3}{2\pi n_s} \left( \frac{T_{e2}}{T_{e1}} \right)^2 = k = \text{constant} \dots \dots \dots (11)$$

$$T_d = \frac{k E_1^2 s R_2}{R_2^2 + s^2 X_{20}^2} \dots \dots \dots (12)$$

Since  $E_1$  is nearly equal to  $V_1$  the equation (12) becomes

$$T_d = \frac{k V_1^2 s R_2}{R_2^2 + s^2 X_{20}^2} \dots \dots \dots (13)$$

The Starting torque is obtained by putting  $s = 1$  in equation (13)

$$T_d = \frac{k V_1^2 R_2}{R_2^2 + X_{20}^2} \dots \dots \dots (14) \text{ or}$$

$$T_d \propto V_1^2 \dots \dots \dots (15)$$

Hence, it is clear from the above equation that the starting torque is proportional to the square of the stator applied voltage.

### Current in terms of flux linkages:

#### What is meant by flux linkage?

In this case the flux linkage is simply the flux density passing through the loop multiplied by its surface area; i.e., . If several turns of the wire are made, this becomes where N is the number of turns. In this case, the surface through which the magnetic flux is passing has been increased by a factor.

Flux linkage is a property of a two-terminal element. Although it is often confused with magnetic flux, the flux linkage is actually an extension rather than an equivalent of magnetic flux. Flux linkage is defined as

$$\lambda = \int \mathcal{E} dt$$

where is the voltage across the device or the potential difference between the two terminals. This definition can also be written in differential form as

$$\mathcal{E} = \frac{d\lambda}{dt}$$

Faraday showed that the magnitude of the EMF generated in a conductor forming a closed loop is proportional to the rate of change of the total magnetic flux passing through the loop (Faraday's law). Thus, for a typical inductance (a coil of conducting wire), the flux linkage is equivalent to magnetic flux, which is the total magnetic field passing through the surface (i.e., normal to that surface) formed by a closed conducting loop coil, and is determined by the number of turns in the coil and the magnetic field; i.e.,

$$\lambda = \int_S \vec{B} \cdot d\vec{S}$$

where is the flux density, or flux per unit area at a given point in space.

The simplest example of such a system is a single circular coil of conductive wire immersed in a magnetic field, in which case the flux linkage is simply the flux passing through the loop.

In order to understand the distinction in terms, it is important to note that the flux through the surface delimited by a coil turn exists independently of the presence of the coil. Furthermore, in a thought experiment with

a coil of turns, where each turn forms a loop with exactly the same boundary, each turn will "link" the "same"

(identically, not merely the same quantity) flux , all for a total flux linkage of . The distinction relies heavily on intuition, and the term "flux linkage" is used mainly in engineering disciplines. Theoretically, the case of a multi-turn induction coil is explained and treated perfectly rigorously with Riemann surfaces: what is called "flux linkage" in engineering is simply the flux passing through the Riemann surface bounded by the coil's turns, hence no particularly useful distinction between flux and "linkage."

Due to the equivalence of flux linkage and total magnetic flux in the case of inductance, it is popularly accepted that the flux linkage is simply an alternative term for total flux, used for convenience in engineering applications. Nevertheless, this is not true, especially for the case of memristor, which is also referred to as the fourth fundamental circuit element. For a memristor, the electric field in the element is not as negligible as for the case of inductance, so the flux linkage is no longer equivalent to magnetic flux. In addition, for a memristor, the flux linkage related energy is dissipated in the form of Joule heating, instead of being stored in magnetic field as done in the case of an inductance: there is no physical magnetic field involved as a link to anything! In conclusion, flux linkage should be interpreted as an extension rather than an equivalent to magnetic flux.

Flux linkage is commonly used to estimate the magnetic flux of an AC motor. However, errors in the voltage measurement and the resulting drift problem of the integral cause the ideal integral to often be replaced with a low-pass filter approximation.



synchronous machines. The theory is now applied in analysis of other types of motors [3-7] including PM synchronous motors, induction motors etc. In Section II, an equivalent 2-phase circuit model of a 3-phase IPM machines is derived in order to clarify the concept of the transformation and the relation between 3-phase quantities and their equivalent 2-phase quantities. Although the above equivalent circuit is very popular, discussions on obtaining parameters of the equivalent circuit for a given motor are rarely found. The main objective of the article is to establish a method to obtain 2-phase circuit parameters from physically measured data. Throughout the article, the following assumptions are made: (1) Stator windings produce sinusoidal mmf distribution. Space harmonics in the air-gaps are neglected. (2) Air-gap reluctance has a constant component as well as a sinusoidally varying component. (3) Balanced 3 phase supply voltage is considered. (4) Although magnetic saturation is considered, eddy current and hysteresis effects are neglected. In addition, presence of damper windings are not considered here because they are not used in PM synchronous machines in general.

### Characterization Of Model Parameters

In order to establish a 2-phase equivalent circuit of a given motor, there are four motor parameters that are to be determined. These are stator resistance  $R_s$ , inductances  $L_q$  and  $L_d$ , and PM flux linkage  $\lambda_m$ . This portion is adapted from [7].

#### (1) Stator resistance $R_s$

Since  $R_s$  is the resistance between line-to-neutral,  $R_s$  would be one half of the measured line-to-line resistance. For most small to medium size PM synchronous motors, skin effect is not significant and can be safely neglected. Winding resistance value is highly temperature-dependent. When winding resistance  $R_o$  is measured, temperature  $T_o$  ( $^{\circ}C$ ) of the winding must be recorded and the resistance  $R_t$  at another temperature  $T$  should be calculated using the following formula.

$$R_t = R_o (K + T) / (K + T_o) \quad (3.1)$$

where  $K$  is the constant determined by the material ( $K=234.5$  for copper). Resistance value at  $25^{\circ}C$  is often used for published data.

#### (2) Synchronous Inductances $L_q$ and $L_d$

To measure inductance, digital RLC meter or inductance bridge is often used. These measurements give inductance value at a small current and may not be suitable to predict characteristics of the motor. The small signal inductance value may be lower than unsaturated inductance value since permeability of iron decreases at very low flux densities [8]. Since inductance is also subject to saturation, we need to measure inductances at various current level of interest. Furthermore, when motor windings are delta-connected or Y connected without neutral line for external access, one can only access line-to-line value. Even when phase inductance value is measured, calculating synchronous inductances based on Eq. 2.24 and 2.25 may not be accurate because we do not know the leakage inductance. In the following, a novel method of directly measuring synchronous inductances  $L_q$  and  $L_d$  with biased current is illustrated.

In order to measure synchronous inductance, we must maintain balanced three phase current condition. Connection diagram of Fig. 3.1 maintains balanced 3 phase current condition at  $\theta = 0^{\circ}$ . When the rotor q-axis (d-axis) is aligned with the center of a-phase winding,  $L_q$  ( $L_d$ ) can be derived from the measured equivalent inductance  $L$  of the circuit in Fig. 3.1, depending on the rotor angle.

$$L_q = (2/3) L(\theta = 0^{\circ}), \quad L_d = (2/3) L(\theta = 90^{\circ}) \quad (3.2)$$

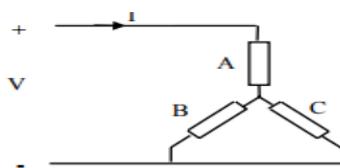


Fig. 3.1 Connection Diagram to measure Synchronous Inductance

This relation can either be proven in a straightforward manner by using Eq. 2.1-2.15, or intuitively as follows. When the rotor is locked at 0 degree (q-axis = a-axis) and supply voltage  $V$  is applied as in Fig. 3.1, Eq. 2.26 becomes

$$V = \{(3/2) R_s + (3/2) L_q p\} I_q \quad (3.3)$$

since  $V_q = (2/3)V$  and  $V_d = 0$ . Since a-phase current  $I$  is the same as  $I_q$ , and the equivalent resistance of the circuit of Fig. 3.1 is  $(3/2) R_s$ , the equivalent inductance seen from the supply source is  $(3/2) L_q$ . Similar explanation can also be applied to  $L_d$  when the rotor is locked at  $90^{\circ}$ .

NEMA [9] suggested two methods of measuring inductance. One is called "biased inductance bridge method". It describes connection diagram when traditional inductance bridge is used. Recently, new digital RLC meters are available. Detailed procedures and circuits to measure inductances with a biased current on these meters differ by manufacturer. Often, special optional devices in addition to the meter unit is necessary. This method, if available, offers a simple and accurate measurement. When the biased inductance measurement is unavailable,

### (3) PM Flux Linkage $\lambda_m$

The flux linkage of the permanent magnet,  $\lambda_m$  can be obtained by measuring the no-load line-to-line rms voltage  $V_{nl}$  of the motor while it is driven through the shaft at a constant speed of  $\omega_m$ . From Eq. 2.26,

$$\lambda_m = \sqrt{(2/3)} V_{nl} / \omega_m \quad (3.5)$$

where  $\omega = (P/2) \omega_m$ . This expression is valid when  $I_q$  and  $I_d$  are negligibly small. It can also be calculated from the manufacturer's data sheet. Since most motor data sheet specify its back-emf constant  $K_b = V_{nl} / \omega_m$ ,

$$\lambda_m = \sqrt{(2/3)} (2/P) K_b. \quad (3.6)$$

Another method of obtaining  $\lambda_m$  is from the measurement of torque and stator current while maintaining orthogonal relationship between rotor and stator magnetic fluxes ( $I_d = 0$ ). Measurement can be done at standstill condition while orthogonality is assured.  $\lambda_m$  can be calculated from

$$\lambda_m = (2/3) (2/P) T / I_s. \quad (3.7)$$

Note that the peak current value (not rms value) has to be used for  $I_s$  in the above equation.. It is convenient to predict armature reaction by plotting measured  $\lambda_m$  values at various current level.

In order to characterize saturation and armature reaction vs current, Frolich's formula may often be used. Assume that  $L_{q0}$ ,  $L_{d0}$  and  $\lambda_{m0}$  be values at linear region where magnitude of  $I_q$  is smaller than  $I_0$ . Since  $L_q$  is subject to saturation and  $L_d$  and  $\lambda_m$  are subject to armature reaction at high currents, the following expressions are often used when magnitude of  $I_q$  is greater than  $I_0$ .

$$L_q(I) = L_{q0} (a + I_0) / (a + |I_q|) \quad (3.8)$$

$$L_d(I) = L_{d0} (b + I_0) / (b + |I_q|) \quad (3.9)$$

$$\lambda_m(I) = \lambda_{m0} (b + I_0) / (b + |I_q|) \quad (3.10)$$

## UNIT V - DYNAMIC ANALYSIS OF SYNCHRONOUS MACHINE

### Dynamic performance of synchronous machine:

AC permanent magnet (PM) motors work as synchronous and brushless DC motors, the motor consists of a wound stator and a rotor. The stator may have a single-phase or multi-phase winding which is sometimes called the armature winding [1] (see Fig. 1). The rotor has a permanent magnet.

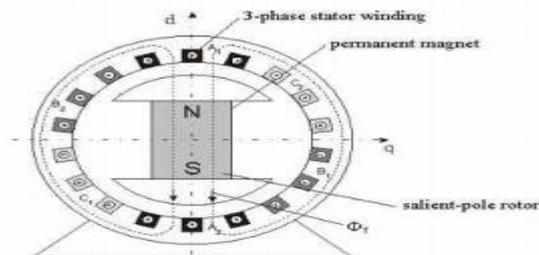
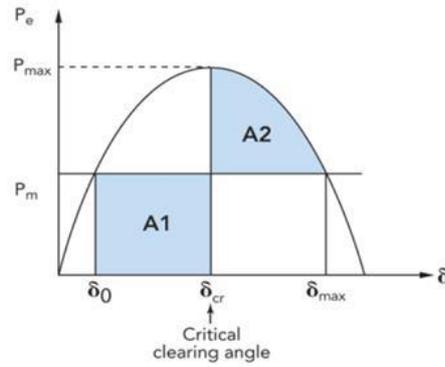


Fig.1.Schematic of three phase PM, salient-pole motor with buried magnet in the rotor

Since there is no winding in the rotor (that would be supplied through the brushes), the AC PM machines are also called brushless PM machines. The rotor of the AC PM motor rotates synchronously with the magnetic field generated by the stator winding. The AC PM motor when operating in each of the above modes, i.e. synchronous or brushless DC motor, performs differently. Thus, its electromechanical characteristics will differ significantly. Especially when these modes of PM motor fed single phase source instead of three phase source. The BLDC motors are available in many different power rating from very small motors as used in hard disk drives to large motor used in electrical vehicles. Three phase motors are most common but two phase motors are also available in many applications. In the reference, the simulation of dynamic state of BLCD motor was presented and the result shown that the torque ripples in three phase system is less when compared to that of two-phase system.

### Equal area criteria

The stability studies which evaluate the impact of disturbances on the behavior of synchronous machines of the power system are of two types: transient stability and steady state stability.

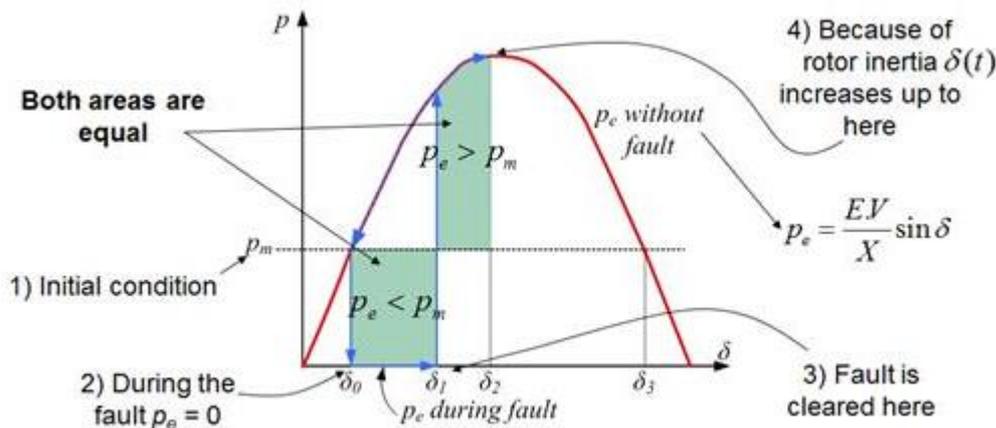


The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to a severe disturbance. This may be a sudden application of large load, a loss of generation, a loss of large load, or a fault (short circuit) on the system. In most disturbances, oscillations are such magnitude that linearization is not permissible and nonlinear equations must be solved to determine the stability of the system.

The steady-state stability is concerned with the system subjected to small disturbances wherein the stability analysis could be done using the linearized version of nonlinear equations. In this experiment we are concerned with the transient stability of power systems.

In a two-machine system, under the usual assumptions of constant input, and constant voltage behind transient reactance, the angle between the machines either increases indefinitely or else, after all disturbances have occurred, oscillates with constant amplitude. In other words the two machines either fall out of step on first swing or never. Under these conditions the observation that the machines come to rest with respect to each other may be taken as the proof that the system is stable. There is a simple graphical method of determining whether the machines come to rest with respect to each other. This method is known as the equal area criterion of stability.

When the fault occurs at  $t=0$ , the electrical power output is suddenly zero while  $p_m$  is unaltered. The difference in power must be accounted for by a rate of change of store kinetic energy in the rotor masses. This is accomplished only by an increase in speed which results from the constant accelerating power  $p_m$ .



After reaching  $\delta_2$ ,  $\delta(t)$  will oscillate until losses and the load damp oscillations and  $\delta(t) = \delta_0$ . If  $\delta_2 > \delta_3$  the generator loses stability because  $p_e < p_m$  and the generator continues to accelerate.